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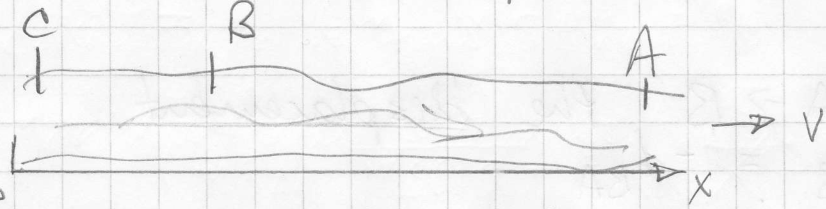
4. To solve this problem, we need to set up a coordinate system and keep careful track of the positive and negative direction of both the velocities & displacements



Velocities

Assume the river is flowing to the right in the positive direction. So, the

velocity of the river = V', relative to the ground.

The swimmer starts swimming upstream at a speed of V relative to the water. So, her speed is

$-V+V'$ relative to ground, and $(A \rightarrow C)$

$V+V'$ relative to the ground when swimming downstream from $C \rightarrow A$.

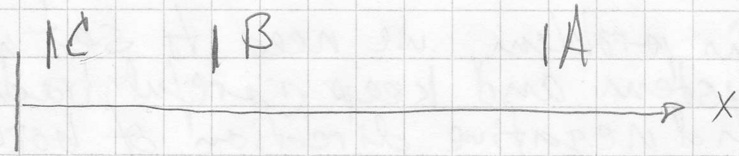
Notice that the velocity upstream is negative. V has to be $> V'$, $V > V'$

& the swimmer is moving to the left, upstream, in the negative x-direction so her velocity must be negative: $-V+V'$

Downstream the two velocities add together in the positive direction: $V+V'$.

4. Displacements

Now lets look at the displacements in the coordinate system.



$d_{BA} = 800m$

$t_{BC} = 18min$

Swimmer

Upstream $A \to B$ the displacement is $d_{AB} = -d_{BA}$

d_{BA} is moving to the right so $d_{BA} > 0$.

$B \to C$: $d_{BC} = -d_{CB}$ & $d_{CB} > 0$.

Downstream displacement is $d_{CA} > 0$.

Log.

Downstream $d_{BA} > 0$.

Now, to do this problem we need to recall that speed (or velocity, more on this later) is distance traveled divided by time. Jus think kilometers per hour $\Rightarrow \frac{km}{hr}$: $\frac{distance}{time}$.

$$V = \frac{d}{t}$$

In this problem, I'm going to work w/ time so solving for t:

$$t = \frac{d}{V}$$

4. cont'd

Now let's consider the time between when the swimmer first met the log at B, and when she catches up to it at A.

Log The log goes downstream with the current at V'.

$$V' = \frac{d_{BA}}{t_{tot}}$$

$$\text{or} \quad t_{tot} = \frac{d_{BA}}{V'}$$

Swimmer

$$t_{tot} = t_{BC} + t_{CB} + t_{BA}$$

$$B \to C \quad -V + V' = \frac{d_{BC}}{t_{BC}} = \frac{-d_{CB}}{t_{BC}}$$

multiply by -1 :
$$V - V' = \frac{d_{CB}}{t_{BC}}$$

solve for t_{BC} :
$$t_{BC} = \frac{d_{CB}}{V - V'}$$

Note $t_{BC} > 0$ as it should. Time flows forward

OR
$$d_{CB} = (V - V')\, t_{BC}$$
(we'll need this later)

$$C \to B \quad V + V' = \frac{d_{CB}}{t_{CB}}$$

$$t_{CB} = \frac{d_{CB}}{V + V'}$$

$t_{CB} > 0$

4. cont'd

$$B \to A \qquad V + V' = \frac{d_{BA}}{t_{BA}}$$

$$t_{BA} = \frac{d_{BA}}{V+V'} \qquad (t_{BA} > 0 \checkmark)$$

Now plugging into the equation of the timer.

$$t_{tot} = t_{BC} + t_{CB} + t_{BA}$$

$$\frac{d_{BA}}{V'} = t_{BC} + \frac{d_{CB}}{V+V'} + \frac{d_{BA}}{V+V'}$$

We know $t_{BC} = 18\,min$ so we keep it. $(= 1080s)$

Move $\frac{d_{BA}}{V+V'}$ over

$$\frac{d_{BA}}{V'} - \frac{d_{BA}}{V+V'} = t_{BC} + \frac{d_{CB}}{V+V'}$$

Replace d_{CB} with $d_{CB} = (V-V')\,t_{BC}$

$$d_{BA}\left(\frac{1}{V'} - \frac{1}{V+V'}\right) = t_{BC} + \frac{V-V'}{V+V'}\,t_{BC}$$

$$d_{BA}\left(\frac{1}{V'} - \frac{1}{V+V'}\right) = t_{BC}\left(1 + \frac{V-V'}{V+V'}\right)$$

Multiply by $V'(V+V')$

$$d_{BA}\left(\frac{(V+V')V'}{\cancel{V'}} - \frac{V'(V+V')}{\cancel{V+V'}}\right) = t_{BC}\left(V'(V+V') + V'(V-V')\right)$$

$$d_{BA}\left(V \cancel{+V'} \cancel{-V'}\right) = t_{BC}\left(V'V \cancel{+V'^2} + V'V \cancel{-V'^2}\right)$$

$$d_{BA}\,\cancel{V} = 2t_{BC} \cdot V'\cancel{V}$$

$$d_{BA} = 2\,t_{BC}\,V'$$

$$V' = \frac{d_{BA}}{2t_{BC}} = \frac{800m}{2(1080s)}$$

$$\boxed{V' = 0.37\,m/s}$$