Ch. 16.4 Dr. Bob

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1. A) True. h gets smaller $\Rightarrow E_{pg} = mgh$ gets smaller.

   B) True. At mid-height half the potential energy is converted to kinetic energy.

   C) True. Mechanical Energy is conserved in a frictionless system.

   D). False: The velocity increases w/ the $\sqrt{E_{pg}}$ lost.

$$\tfrac{1}{2} m v^2 = \Delta E_{pg}$$

$$v = \sqrt{\frac{2\,\Delta E_{pg}}{m}}$$

2.

$m = 1.00\,kg$

a) $E_{pgi} = mgh_i = (1.00\,kg)(9.8\,m/s^2)(2\,m)$

$$\boxed{E_{pgi} = 19.6\ J}$$

b) $E_{ki} = 0\ J$

c) $E_{pgf} = 0\ J$

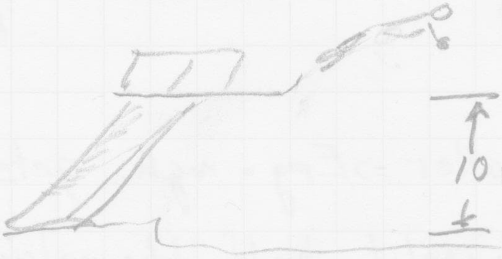
d) $E_{kf} = E_{pgi} = 19.6\ J$ $\boxed{E_{kf} = 19.6\ J}$

e) $E_{kf} = \tfrac{1}{2} m V_f^2 = E_{pgi}$

$$V_f = \sqrt{\frac{2\,E_{pgi}}{m}} = \sqrt{\frac{2(19.6\ J)}{1.00\,kg}} = 6.26\,m/s$$

$$\boxed{V_f = 6.26\,m/s}$$

3.



$h_i = 10.0m$

$h_f = 0m$

$V_i = 4m/s$

$V_f = ?$

$$E_{mi} = \frac{1}{2} m V_i^2 + mgh_i$$

$$E_{mf} = \frac{1}{2} m V_f^2 + mg\cancel{h_f}^{\to 0}$$

Conservation of Energy.

$$E_{mf} = E_{mi}$$

$$\frac{1}{2}\cancel{m} V_f^2 = \frac{1}{2}\cancel{m} V_i^2 + \cancel{m}gh_i$$

$$V_f = \sqrt{V_i^2 + 2gh_i}$$

$$= \sqrt{(4m/s)^2 + 2(9.8m/s^2)(10.0m)}$$

$$\boxed{V_f = 14.6 \, m/s}$$

Interesting that this is true independent of the mass of the diver. This comes from the fact that all objects experience an acceleration of 9.8m/s² in gravity on Earth. Remember the bowling ball & the feathers.

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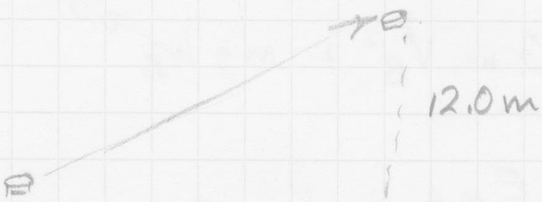
4.



$h_i = 0m$
$h_f = 12m$

$V_i = 25m/s$

$$E_{mi} = \tfrac{1}{2}mV_i^2 + \cancelto{0}{mgh_i}$$

$$E_{mf} = \tfrac{1}{2}mV_f^2 + mgh_f$$

$$E_{mf} = E_{mi}$$

$$\tfrac{1}{2}\cancel{m}V_f^2 + \cancel{m}gh_f = \tfrac{1}{2}\cancel{m}V_i^2$$

$$\tfrac{1}{2}V_f^2 = \tfrac{1}{2}V_i^2 - gh_f$$

$$V_f = \sqrt{V_i^2 - 2gh_f}$$

$$V_f = \sqrt{(25m/s)^2 - 2(9.8m/s^2)(12.0m)}$$

$$\boxed{V_f = 19.7\,m/s}$$

5.

$m = 90kg$

$h_A = 10m$ $\quad$ $h_B = 0m$ $\quad$ $h_C = 3.0m$

$V_A = 0m/s$ $\quad$ $V_B = ?$ $\quad$ $V_C = ?$

$$a)\; E_{mA} = \tfrac{1}{2}m\cancelto{0}{V_A^2} + mgh_A$$

$$= (90kg)(9.8m/s^2)(10m)$$

$$\boxed{E_{mA} = 8{,}820\,J}$$

b) $E_{mA} = E_{mB} = 8820\,J$ $\quad V_B = ?$

$$E_{mB} = \frac{1}{2} m V_B^2 + mg\cancelto{0}{h_B}$$

$$\frac{1}{2} m V_B^2 = E_{mA}$$

$$V_B = \sqrt{\frac{2E_{mA}}{m}} = \sqrt{\frac{2(8820J)}{90kg}}$$

$$\boxed{V_B = 14\,m/s}$$

c) $E_{mc} = E_{mA} = 8820\,J$ $\quad V_c = ?$

$$\frac{1}{2} m V_c^2 + mgh_c = E_{mA}$$

$$\frac{1}{2} m V_c^2 = E_{mA} - mgh_c$$

$$V_c = \sqrt{\frac{2(E_{mA} - mgh_c)}{m}}$$

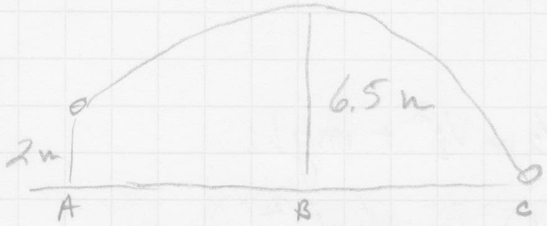
$$= \sqrt{\frac{2(8820J - (90kg)(9.8m/s^2)(3.0m))}{90kg}}$$

$$\boxed{V_c = 11.7\,m/s}$$

P. 353 $m = 4.00 kg$ Dr. Bob

6.

$V_i = 14 m/s$
$h_i = 2m$



$$E_{mi} = \frac{1}{2} m V_i^2 + mgh_i$$

$$= \frac{1}{2}(4.00kg)(14 m/s)^2 + (4.00kg)(9.8 m/s^2)(2m)$$

$$E_{mi} = 470.4 J$$

a) $E_{mB} = E_{mi} = 470.4J$ $h_B = 6.5m$

$$\frac{1}{2} m V_B^2 + mgh_B = E_{mi}$$

$$\frac{1}{2} m V_B^2 = E_{mi} - mgh_B$$

$$V_B^2 = \frac{2(E_{mi} - mgh_B)}{m}$$

$$V_B = \sqrt{\frac{2(E_{mi} - mgh_B)}{m}}$$

$$= \sqrt{\frac{2(470.4J - (4.00kg)(9.8 m/s^2)(6.5 m))}{(4.00kg)}}$$

$$\boxed{V_B = 10.4 m/s}$$

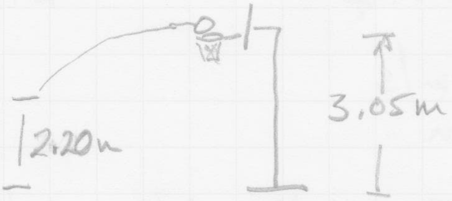
b) $E_{mc} = E_{mi} = 470.4J$ $h_c = 0m$

$$\frac{1}{2} m V_c^2 + \cancelto{0}{mgh_c} = E_{mi}$$

$$V_c = \sqrt{\frac{2E_{mi}}{m}} = \sqrt{\frac{2(470.4J)}{4.00kg}}$$

$$\boxed{V_c = 15.3 m/s}$$

7.



$V_f = 8.00\,m/s$ $\quad$ $h_f = 3.05\,m$

$V_i = ?$ $\quad$ $h_i = 2.20\,m$

$$E_{mf} = E_{mi}$$

$$\frac{1}{2}\cancel{m}V_f^2 + \cancel{m}gh_f = \frac{1}{2}\cancel{m}V_i^2 + \cancel{m}gh_i$$

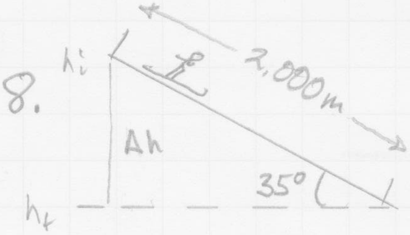
$$\frac{1}{2}v_f^2 + g(h_f - h_i) = \frac{1}{2}v_i^2$$

$$V_i^2 = V_f^2 + 2g(h_f - h_i)$$ ← Look familiar?

$$V_i = \sqrt{(8.00\,m/s)^2 + 2(9.8\,m/s^2)(3.05\,m - 2.20\,m)}$$

$$\boxed{V_i = 8.93\,m/s}$$

8.



$V_i = 0\,m/s$ $\quad$ $V_f = ?$

$h_i = (2000\,m)\sin 35° = 1147\,m$

$h_f = 0\,m$

$$E_{mi} = E_{mf}$$

$$\frac{1}{2}\cancel{m}\cancel{v_i^2}^{\to 0} + \cancel{m}gh_i = \frac{1}{2}\cancel{m}V_f^2 + \cancel{mgh_f}^{\to 0}$$

$$mgh_i = \frac{1}{2}V_f^2$$

$$V_f = \sqrt{2gh_i}$$

$$= \sqrt{2(9.8\,m/s^2)(2000\,m)\sin 35°}$$

$$\boxed{V_f = 149.9\,m/s}$$

or 540km/h!

but wind resistance
does exist...

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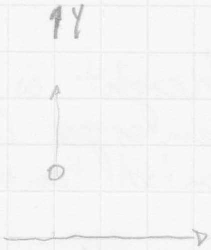
9.



$V_i = 20.0\,m/s \quad h_i = 0\,m$

$m = 0.200\,kg$

a) When does $\frac{1}{2} m V^2 = mgh$?

$$E_{mi} = \frac{1}{2} m V_i^2 + mg\cancel{h_i}^{\,0}$$

$$= \frac{1}{2}(0.200\,kg)(20.0\,m/s)^2$$

$$E_{mi} = 40.0\ J$$

$E_k = E_{pg}$ when they each have ½ the total energy

$mgh = \frac{1}{2}(40J) \Rightarrow$ half total E_m

$$h = \frac{20J}{mg} = \frac{20J}{(0.200kg)(9.8m/s^2)}$$

$$\boxed{h_f = 10.2m}$$

b) When does $E_k = \frac{1}{4} E_{pg}$ $\quad E_{pg} = 4E_k$

$$E_k + E_{pg} = E_{mi} \text{ (or total)}$$

$$\frac{1}{4}E_{pg} + E_{pg} = E_{mi}$$

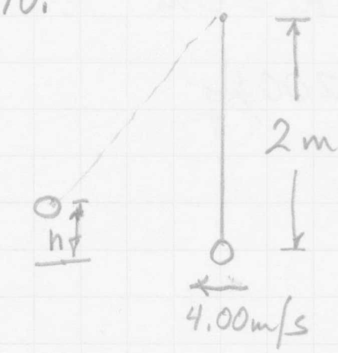
$$\frac{5}{4}E_{pg} = E_{mi}$$

$$E_{pg} = \frac{4}{5}E_{mi} = \frac{4}{5}(40J) = 32J$$

$$h_f = \frac{32J}{mg} = \frac{32J}{(0.200kg)(9.8m/s^2)}$$

$$\boxed{h_f\ 16.3\,m}$$

10.



The maximum height is when all the kinetic energy is converted to potential energy.

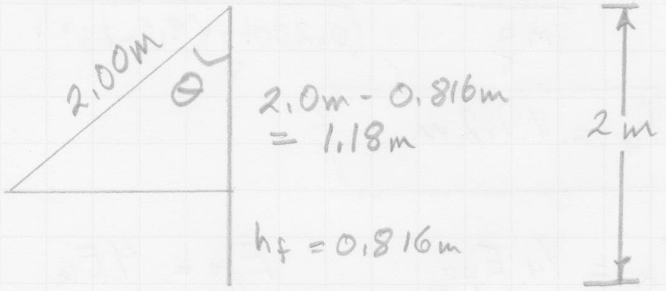
$E_{pgf} = E_{ki}$

$v_i = 4.00 m/s$
$h_i = 0 m$

$$m g h_f = 1/2 m v_i^2$$

$$h_f = \frac{v_i^2}{2g} = \frac{(4.00 m/s)^2}{2(9.8 m/s^2)} = 0.816 m$$

$$h_f = 0.816 m$$



$$\cos\theta = \frac{1.18 m}{2.00 m}$$

$$\theta = \cos^{-1}\left(\frac{1.18 m}{2.00 m}\right)$$

$$\boxed{\theta = 53.7°}$$