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1. a) Point A E_{pg} is greatest
   Point D E_{pg} is smallest

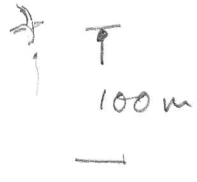
   b) Point D E_k is greatest
   Point A E_k is smallest

   c) With no loss of mechanical energy due to friction, the mechanical energy is equal at point A & at point D.

2. a) With no friction, the marbles will have the same velocity at the bottom.

   b) For the most part, the ball on slide Y would reach the end first. This is because the ball accelerates to a higher velocity early in the transit & this makes up for the longer distance traveled. For a detailed analysis of the problem see the work at the end of this problem set.

3.



$V_{xi} = 30\,km/h = 8.\bar{3}\,m/s$
$V_{xf} = V_{xi}$
$V_{iy} = 0\,m/s$
$V_{fy} = ?$

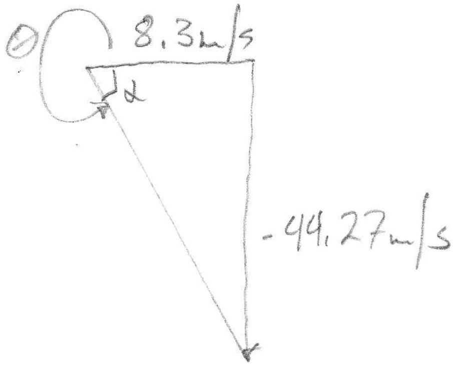
$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}V_{fy}^2$$

$$V_{fy} = \sqrt{2gh}$$

$$V_{fy} = \sqrt{2(9.8\,m/s^2)(100\,m)}$$

$$V_{fy} = 44.27\,m/s$$

$$V_f = \sqrt{V_{xf}^2 + V_{yf}^2} = \left[(8.3\,m/s)^2 + (44.27\,m/s)^2\right]^{1/2} = 45\,m/s$$

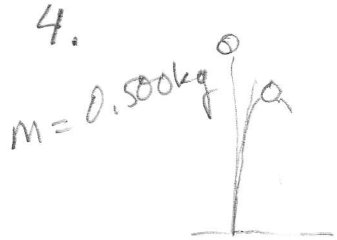


$$\alpha = \tan^{-1}\left(\frac{-44.27\,m/s}{8.33\,m/s}\right) = -79.3^\circ$$

$$\theta = 280.7^\circ$$

$$\boxed{\vec{V}_f = 45\,m/s\ @\ 281^\circ}$$

4\.



The difference in potential energy from the initial height (2.00m) and the top of the first bounce (1.50m) is the heat energy dissipated.

$$mgh_i + \cancel{\tfrac{1}{2}mv_i^2} = mgh_f + \cancel{\tfrac{1}{2}mV_f^2} + E_{th}$$

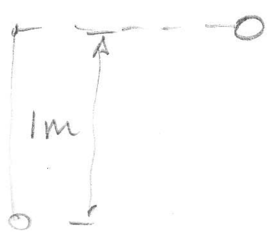
$$E_{th} = mgh_i - mgh_f$$

$$= mg(h_i - h_f)$$

$$= (0.500kg)(9.8m/s^2)(2.00m - 1.50m)$$

$$\boxed{E_{th} = 2.45\,J}$$

5\.



$$mgh = \tfrac{1}{2}\cancel{m}V^2$$

$$V^2 = 2gh$$

$$V = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(1m)}$$

$$\boxed{V = 4.43\,m/s}$$

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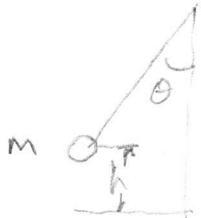


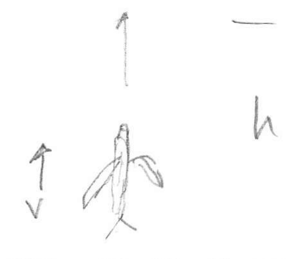
The maximum velocity doesn't depend on the mass.

The velocity is maximum at the bottom of the arc. The potential energy is completely converted to kinetic energy.

$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}V^2$$ The mass cancels out.

$$V = \sqrt{2gh}$$

7.



$V_i = 10\,m/s$ $\quad V_f = 0\,m/s$

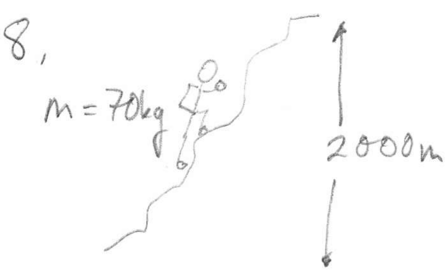
$h_i = 0\,m$ $\quad h_f = ?$

$$\cancel{m}gh_f = \tfrac{1}{2}\cancel{m}V_i^2$$

$$h = \frac{V_i^2}{2g} = \frac{(10\,m/s)^2}{2(9.8\,m/s^2)}$$

$$\boxed{h = 5.1\,m}$$

8.



a) Increase in $E_{pg} = mgh$

$$\Delta E_{pg} = (70\,kg)(9.8\,m/s^2)(2000\,m)$$

$$\boxed{\Delta E_{pg} = 1.372\times10^6\,J}$$

b) Total chemical Energy used, $E_c = \dfrac{\Delta E_{pg}}{0.25} = 5.488\times10^6\,J$

$$E_c = 5.488\times10^6\,J\left(\frac{1\,g\,fat}{3.0\times10^4\,J}\right)$$

$$\boxed{E_c = 183\,g\,fat}$$

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Problem 2 Revisit in greater depth.

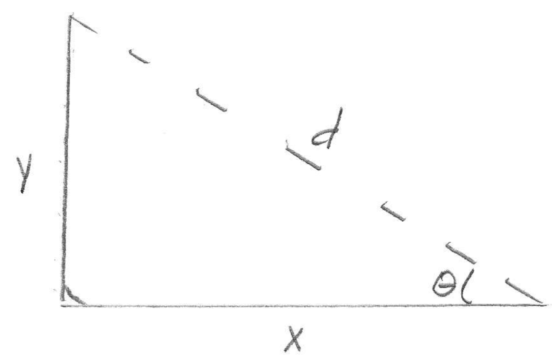
Look at two cases.

Case I - The ramp is curve such that the ball falls straight down almost to the bottom, then travels horizontally to the end.

Case II - The marble goes straight to end as in slide X on p. 361.

Method: Calculate the time it takes to travel path & compare the times. To take into account steep & less steep paths in Case II, determine the times in terms of the ramp angle θ.

Case I



d - length of ramp.

$y = d \sin\theta$

$x = d \cos\theta$

$V_i = 0\,m/s$

Marble drops straight down:

$$mgy = \tfrac{1}{2} m V^2$$

$$V^2 = 2gy$$

$$V_f = \sqrt{2gy}$$

(This is also the final velocity of Case II)

Case I

The time to drop:

$$V_f = \cancel{V_i}^{0} + a\,\Delta t$$

$$\Delta t_1 = \frac{V_f}{g}$$

$$\Delta t_1 = \frac{\sqrt{2gy}}{g} = \sqrt{\frac{2y}{g}}$$

Time to travel horizontally:

$$d = V \cdot \Delta t$$

$$\Delta t_2 = \frac{X}{V_f} = \frac{X}{\sqrt{2gy}}$$

Total time Case I.

$$\Delta t_{tot} = \Delta t_1 + \Delta t_2$$

$$\Delta t_{tot} = \sqrt{\frac{2y}{g}} + \frac{x}{\sqrt{2gy}}$$

Now write the total in terms of θ.

$$\Delta t_{tot} = \sqrt{\frac{2\,d\sin\theta}{g}} + \frac{d\cos\theta}{\sqrt{2gd\sin\theta}}$$

$$= \sqrt{\frac{2d}{g}}\sqrt{\sin\theta} + \sqrt{\frac{2d^2}{2^2 g d}}\;\frac{\cos\theta}{\sqrt{\sin\theta}}$$

$$= \sqrt{\frac{2d}{g}} \cdot \sqrt{\sin\theta} + \sqrt{\frac{2d}{g}} \cdot \frac{1}{2} \cdot \frac{\cos\theta}{\sqrt{\sin\theta}}$$

Case I total time:

$$\Delta t = \sqrt{\frac{2d}{g}}\left(\sqrt{\sin\theta} + \frac{1}{2}\frac{\cos\theta}{\sqrt{\sin\theta}}\right)$$

<u>Cas II</u> $V_f = \sqrt{2gy}$

$$X_f = X_i + \frac{1}{2}\left(\cancel{V_i}^{\,0} + V_f\right)\Delta t$$

$$X_f - X_i = \frac{1}{2} V_f \Delta t$$

$$\Delta X = \frac{1}{2} V_f \Delta t \qquad \Delta x = d$$

$$2d = V_f \Delta t$$

$$\Delta t = \frac{2d}{V_f}$$

$$\Delta t_2 = \frac{2d}{\sqrt{2gy}} = \frac{2d}{\sqrt{2gd\sin\theta}}$$

$$\Delta t_2 = \sqrt{\frac{2^{\cancel{2}} d^{\cancel{2}}}{2g\cancel{d}}}\ \frac{1}{\sqrt{\sin\theta}}$$

$$\Delta t_2 = \sqrt{\frac{2d}{g}} \cdot \frac{1}{\sqrt{\sin\theta}}$$

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Problem 2 (In detail cont'd)

Now subtract the times for the two paths & plot.

$\Delta T =$ Path I $-$ Path II time.

If ΔT is negative then path II takes longer like what we expect.

If ΔT is positive then path I takes longer

$$\Delta T = \Delta t_1 - \Delta t_2$$

$$= \sqrt{\frac{2d}{g}}\left[\sqrt{\sin\theta} + \frac{1}{2}\frac{\cos\theta}{\sqrt{\sin\theta}} - \frac{1}{\sqrt{\sin\theta}}\right]$$

$$= \sqrt{\frac{2d}{g}} \cdot \frac{1}{\sqrt{\sin\theta}}\left[\sin\theta + \frac{1}{2}\cos\theta - 1\right]$$

For $0 < \theta \leq 90°$, $\sqrt{\sin\theta} > 0$.

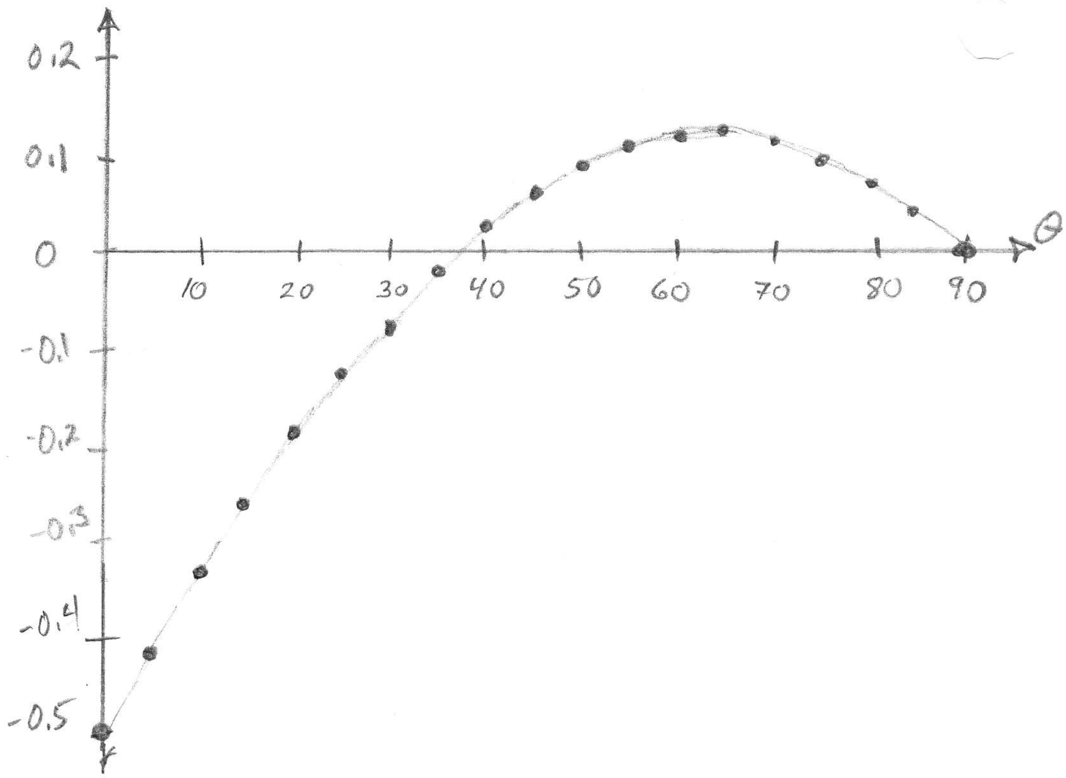
so $\sqrt{\frac{2d}{g}} \cdot \frac{1}{\sqrt{\sin\theta}} > 0$.

We only need to plot

$$F(\theta) = \sin\theta + \tfrac{1}{2}\cos\theta - 1$$

Angle	Function
0	-0.500
5	-0.415
10	-0.334
15	-0.258
20	-0.188
25	-0.124
30	-0.067
35	-0.017
40	0.026
45	0.061
50	0.087
55	0.106
60	0.116
65	0.118
70	0.111
75	0.095
80	0.072
85	0.040
90	0.000

Use Excel to calculate the data to plot

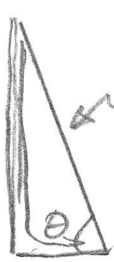


So for angles of $\theta \lesssim 37.5^\circ$, slide Y is faster

These ramps have a relatively small angle.

Faster.

For angles, $37.5^\circ < \theta \leq 90^\circ$, slide Y is faster than the steeply curved ramp.



Faster