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Ch 17

Dr. Bob

1. a) $k = 2000\,N/m$

$\Delta X = 0.400m$

$E_e = \frac{1}{2} k \Delta X^2$

$= \frac{1}{2}(2000\,N/m)(0.400m)^2$

$\boxed{E_e = 160J}$

b)

$E_e = \frac{1}{2} m V^2$

$m = 2kg$

$V = \sqrt{\dfrac{2E_e}{m}} = \sqrt{\dfrac{2(160J)}{(2kg)}} = \boxed{12.6\,m/s}$

Units: $\sqrt{\dfrac{J}{kg}} = \sqrt{\dfrac{kg \cdot m^2/s^2}{kg}} = \sqrt{m^2/s^2} = m/s$

2. a) $k = 650\,N/m$

$\Delta X = 0.100m$

$E_e = \frac{1}{2} m \Delta X^2 = \frac{1}{2}(650\,N/m)(0.100m)^2$

$\boxed{E_e = 3.25J}$

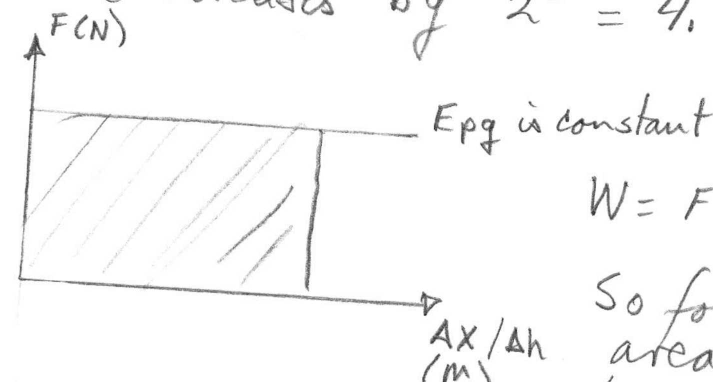
b) $3E_e = \frac{1}{2} k \Delta X^2$

$\Delta X = \sqrt{\dfrac{6E_e}{k}} = \sqrt{\dfrac{6(3.25J)}{650\,N/m}} = 0.173m$

Additional length $0.173m - 0.100m = \boxed{0.073m}$

3. a) since $E_e = \frac{1}{2} k \Delta X^2$, when ΔX doubles E_e increases by $2^2 = 4$.

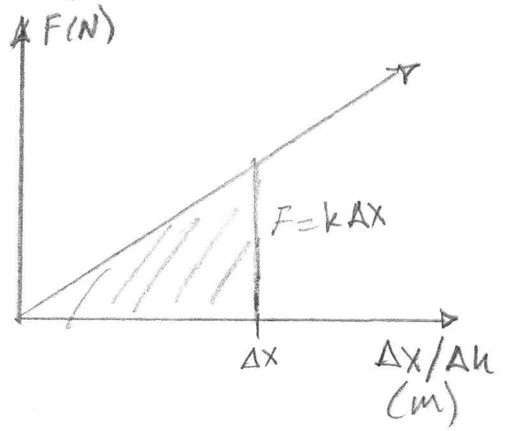
b)



$W = F \cdot \Delta X$; $N \cdot m = J$

So for gravity the area (= Energy) increases linearly w/ ΔX!

3. b) (cont'd)



$W = F \cdot \Delta X$

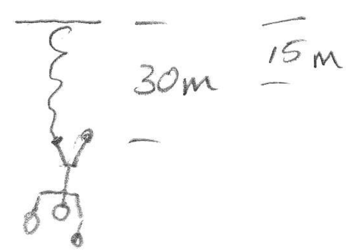
Here F increases w/ ΔX.

The area (Energy) increases as the area of a triangle increases

The area is $\frac{b \cdot h}{2}$

$$W = \frac{(F)\Delta X}{2} = \frac{(k\Delta X)(\Delta X)}{2} = \frac{1}{2} k \Delta X^2$$

4. $m = 65.0\,kg$



$h = 30m$

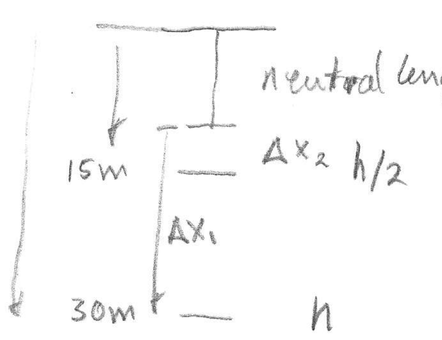
Gravitational potential energy $E_{pg} = mgh$
equals elastic potential energy $E_e = \frac{1}{2} k \Delta X^2$

$$mgh = \frac{1}{2} k \Delta X_1^2$$

At 15m

$\Sigma F_y = 0; \quad F_e = F_g; \quad k\Delta X_2 = mg$

(Way too complex for this course. Go to next problem.)



neutral length of cord, L

$L + \Delta X_2 = h/2 \qquad L = h/2 - \Delta X_2$

$L + \Delta X_1 = h \qquad L = h - \Delta X_1$

$15m - \Delta X_2 = h - \Delta X_1$

4. (cont'd)

$$\Delta X_1 - \Delta X_2 = h/2$$

$$\Delta X_2 = \Delta X_1 - h/2$$

From the force equation

$$k\,\Delta X_2 = mg$$

$$k(\Delta X_1 - h/2) = mg$$

$$\Delta X_1 - h/2 = \frac{mg}{k} \qquad \Delta X_1 = \frac{mg}{k} - \frac{h}{2}$$

From the energy equation.

$$mgh = \tfrac{1}{2} k \left(\frac{mg}{k} - \frac{h}{2} \right)^2$$

$$mgh = \frac{k}{2} \left(\frac{m^2 g^2}{k^2} - (h)\frac{mg}{k} + \frac{h^2}{4} \right)$$

$$h = \frac{k}{2} \left(mg - \frac{h}{k} + \frac{h^2}{4mg} \right)$$

Divide by mg

$$h = \frac{mg}{2k} - \frac{h}{2} + \frac{h^2}{8mg}\,k$$

$$0 = \frac{mg}{2k} - \frac{3h}{2} + \frac{h^2}{8mg}\,k$$

Factor out $\frac{1}{2k}$

$$\frac{1}{2k}\left(mg - (3h)k + \frac{h^2}{4mg}\,k^2 \right) = 0$$

For this to be true the quadratic eq'n must be true.

$$\left(\frac{h^2}{4mg}\right) k^2 - (3h)k + mg = 0$$

$$\left(\frac{(30\,m)^2}{4(65kg)9.8\,m/s^2)}\right)k^2 - (90m)k + (65kg)(9.8\,m/s^2) = 0$$

$$\left(0.3532\,\frac{m^2}{N}\right)k^2 - (90m)k + 637N = 0$$

Use quadratic equation calculator online...

$$k = 247.5\,\frac{N}{m} \quad \text{or} \quad 7.286\,N/m$$

Choose $\boxed{k = 247.5\,N/m}$

If $k = 7.286\,N/m$ $\quad \Delta X_2 = \frac{mg}{k} = \frac{(65kg)(9.8\,m/s^2)}{7.286\,N/m}$

$\Delta X_2 = 87.4\,m$ Not possible

But if $k = 247.5\,N/m$ $\quad \Delta X_2 = \frac{(65kg)(9.8\,m/s^2)}{247.5\,N/M}$

$\Delta X_2 = 2.57\,m$

this is reasonable.

The solution I saw online did not take into account $\Delta X_2 > 0$. They set $\Delta X_2 = 0$

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5. a) Slope $k = \frac{50N}{0.8m}$ $\boxed{K = 62.5 \frac{N}{m}}$

b) $F \cdot \Delta x = N \cdot m = J$

6. $\Delta x = 0.35m$
$F_e = 10.5N$

a) $k = \frac{F}{\Delta x} = \frac{10.5N}{0.35m} = 30 N/m$

$E_e = \frac{1}{2} k \Delta x^2 = \frac{1}{2}(30 N/m)(0.35m)^2$

$\boxed{E_e = 1.84 J}$

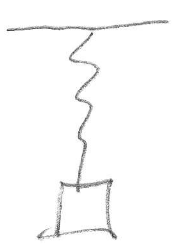
b) $\Delta x = 0.20m$

$E_e = \frac{1}{2} k \Delta x^2 = \frac{1}{2}(30 N/m)(0.20m)^2 = 0.6J$

$\Delta E_e = 0.6J - 1.84J = -1.24J$

$\boxed{\Delta E_e = -1.24J}$ 1.24J less

7.



$m = 0.20kg$
$k = 55 N/m$

Answer b) first.

b) $V_i = 0 m/s$ $V_f = 0 m/s$ Use conservation of Mechanical Energy.
$h_i = ?$ $h_f = 0m$
$\Delta x_i = 0m$ $\Delta x_f = h_i$

$\cancel{\frac{1}{2} m V_i^2}^{0} + mgh_i + \cancel{\frac{1}{2} k \Delta x_i^2}^{0} = \cancel{\frac{1}{2} m V_f^2}^{0} + \cancel{mgh_f}^{0} + \frac{1}{2} k \Delta x_f^2$

$$mgh_i = \frac{1}{2}k\,\Delta x_i^2 = \frac{1}{2}k \cdot h_i^2$$

$$mg = \frac{1}{2}k\,h_i$$

$$h_i = \frac{2mg}{k} = \frac{2(0.2\,kg)(9.8\,m/s^2)}{55\,N/m}$$

$$h_i = 0.0713\,m$$

a)

$\Delta x = 0\,m$ — $0.073\,m$

$0.0563\,m$] $0.015\,m$

$h = 0\,m$

$V_i = 0\,m/s$ $\quad$ $h_i = 0.0713\,m$ $\quad$ $\Delta x_i = 0\,m$

$V_f = ?$ $\quad$ $h_f = 0.0563\,m$ $\quad$ $\Delta x_f = 0.015\,m$

$$\cancel{\frac{1}{2}mV_i^2}^{\,0} + mgh_i + \cancel{\frac{1}{2}k\,\Delta x_i^2}^{\,0} = \frac{1}{2}mV_f^2 + mgh_f + \frac{1}{2}k\,\Delta x_f^2$$

$$mg(h_i - h_f) - \frac{1}{2}k\,\Delta x_f^2 = \frac{1}{2}mV_f^2$$

$$2g(h_i - h_f) - \frac{k}{m}\,\Delta x_f^2 = V_f^2$$

$$V_f = \left[2g(h_i - h_f) - \frac{k}{m}\,\Delta x_f^2\right]^{1/2}$$

$$= \left[2(9.8\,m/s^2)(0.0713\,m - 0.0563\,m) - \frac{55\,N/m}{(0.20\,kg)}(0.015\,m)^2\right]^{1/2}$$

$$\boxed{V_f = 0.48\,m/s}$$

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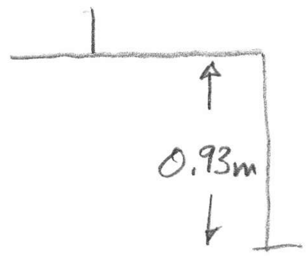
8. $k = 12\ N/m$

$m = 8.3 \times 10^{-3}\ kg$

$\Delta X = 0.040\ m$



$\Delta y = 0.93\ m$

Part I Find $V_i = ?$

$$\tfrac{1}{2} k \Delta X^2 = \tfrac{1}{2} m V^2$$

$$V^2 = \frac{k}{m} \Delta X^2$$

$$V = \sqrt{\frac{k}{m} \Delta X^2} = \sqrt{\frac{(12\ N/m)(0.040\ m)^2}{8.3 \times 10^{-3}\ kg}}$$

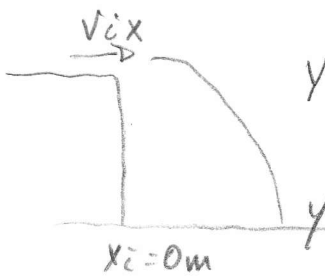
$V = 1.5\ m/s$

Units:

$$\sqrt{\frac{(N/m)\ m^2}{kg}} = \sqrt{\frac{(\cancel{kg}\cdot m/s^2)(\cancel{m^{-1}})(m^2)}{\cancel{kg}}}$$

$$= \sqrt{m^2/s^2} = m/s$$

Part II Now its a UAM. equation problem, projectile motion, with the ball launched horizontally.



$y_i = 0.93\ m$ $V_{ix} = 1.5\ m/s$ $a_y = -9.8\ m/s^2$

$V_{iy} = 0\ m/s$

$y_f = 0\ m$

Find how long it takes to fall in free-fall.

$$\overset{0}{\cancel{y_f}} = y_i + \overset{0}{\cancel{V_{iy} \Delta t}} + \tfrac{1}{2} a_y \Delta t^2$$

$$-2y_i = a_y \Delta t^2$$

$$\Delta t = \sqrt{\frac{-2y_i}{a_y}} = \sqrt{\frac{-2(0.93m)}{-9.8m/s^2}}$$

$$\Delta t = 0.436\ s$$

Now how far in x-direction,

$$X_f = \cancel{X_i}^{\,0} + V_{ix}\,\Delta t$$

$$X_f = (1.5m/s)(0.436\ s)$$

$$\boxed{X_f = 0.65m}$$